Application of Markov Chain (MC) Model to the Stochastic Forecasting of Stocks Prices in Nigeria: The Case Study of Dangote Cement

AGBAM, Azubuike Samuel (F,cfia; F,cilrm)
Department of Banking and Finance,
Faculty of Management Sciences,
Rivers State University,
Nkpolu-Oroworukwo, Port Harcourt,
NIGERIA.
Phone: 0706 738 1059
Email azubuikesamuelagbam@yahoo.com

Udo, Ephraim Okon
Department of Mathematics/Statistics,
Faculty of Science,
Rivers State University,
Nkpolu-Oroworukwo, Port Harcourt,
NIGERIA.
Phone: 0803 880 5513
Email ephraimokon@rocketmail.com

Abstract

To counter strong features of disorder and randomness of stock market fluctuation in Nigeria, this paper introduces a Markov process models for the stock market trend forecasting, which is a useful complement for an existing technical analysis. An understanding of the stock market trend in terms of predicting price movements is important for investment decisions. Markov chain model has been widely applied in predicting stock market trend. In many applications, it has been applied in predicting stock index for a group of stock but little has been done for a single stock. The overall objective of this study is to apply Markov chain to model and forecast trend of Dangote Cement shares trading in the Nigerian Stock Exchange. The study was conducted through longitudinal case study design. Secondary quantitative data on the daily closing shares prices was obtained from the Nigerian Stock Exchange website over a period covering 1st January, 2018 to 31st December, 2019 forming 464 days trading data panel. A Markov chain model was determined based on probability transition matrix and initial state vector. In the long run, irrespective of the current state of shares prices, the model predicted that the Dangote Cement would depreciate, maintain and appreciate respectively.

Keywords: *Markov chains, Markov process, transition probability matrix, stock index, stock market, trend prediction, Dangote Cement, Nigeria.*

1. Introduction

Decision making plays a very important role on individual, organizational, societal and governmental levels. The decision maker (investor), after considering all surrounding circumstances, has to go through the mental process before an action is taken among several alternatives. The kind of decision taken by the investor today affects its future either positively or negatively. The fundamental decision faced by the investor is how to optimally allocate its funds at each decision epoch during a time horizon on an uncertain market environment in order to optimize its reward.

Traditionally, investment is the current commitment of resources in order to achieve later benefits. These benefits are obtained under portfolio management which is a decision process of dividing the total investment funds among some major asset classes such as equities, bonds, goods etc (Haleh, 2009). Portfolio allocation is how an investor allocates his funds among a set of investments to maximize return while simultaneously minimizing risk (David, 2008). A portfolio is constructed with the aim of achieving a maximum expected return for a given risk and time horizon. Portfolio allocation problem is a problem which has generated a great deal of research since it was first formally defined by Markowitz (1952), where he used mean variance (MV) optimization model as a breakthrough achievement in modern portfolio theory (David, 2008).

The stock market has been a significant contributor in a swiftly growing economy. In the context of capital formation, stock market is one of the best alternative for various business houses and companies for further expansion or setting up of a new venture. Generally, stocks are the shares of company or organization. The stock exchange is a legal framework where an individual or group of individual can buy and sell such shares in a systematic way. In other words, stock exchange is the meeting place of both buyers and sellers (wider domain of trading activities) of stocks.

The sole objective of investors to purchase stock is motivated by the desire for capital appreciation. Generally the companies making more profit offer greater return to the investors than those companies making less profit or no profit. The share price of the companies depends on the performance of the companies.

In this regard, the return on investment made by individuals, corporate bodies or organizations in the stock market depends on the choice or decision of selecting appropriate companies to purchase stocks. We most likely find investors seeking to know the background and historical behavior of listed equities to assist investment decision making. The decision on selecting the most beneficial options in the stock market is extremely depends on how well informed you are in the stock analysis. That is why it is most essential to come up with statistical models and their analysis. These models help to predict the share price movement of stocks. The random fluctuation of price of shares causes the uniform distribution of market information. This inherent stochastic behavior of stock market makes the prediction of possible states of the market more complicated.

To the financial analysts, understanding how price behaves is of great importance; and also to use the knowledge of price behaviour to reduce risk or take better and well informed decisions about the future states of the price. Building a model, which gives a detailed description of how successive prices would be, might be desirable. Furthermore, the inherent uncertainty associated with financial time series, given the complexity involved in its application has made the study and analysis subject of concern to statisticians and economists (Tsay, 2005), Shinha et al., (2010), Chakabarti et al., (2006). Hence, analyzing financial data using appropriate, if possible, new models, is of interest to market participants (Chakrabarti, Chakraborti and Chatterjee, 2006).

Several statistical approaches have been employed to predict the behaviour of stocks. These range from artificial neural networks (ANN) [Neewi, Asagba and Kabari, 2013], Data mining and Regression [Abdulsalam, Adewole and Jimoh, 2011], autoregressive integrated moving average (ARIMA) [Adebiyi, Adewunmi and Ayo, 2014], weighted Markov chain [Qing-Xin, 2014], to Markov chain analysis. Markov chain test is applied to individual stock prices as well as market indices at various frequencies for different time periods; see for example [Nierderhoffer and Osborne, 1966, Fieltz, 1971; Fieltz, 1975] and [McQueen and Thorley, 1991]. [Fieltz and Bhargava, 1973] specifically tested for the stationarity (or time homogeneity) as well as the order of dependence of the Markov Chain using daily and weekly returns of 200 individual stocks from 1963 to 1968 in a 3-state Markov Chain. Bachelier (1914) looked at fluctuations in stock prices as a random walk; Fama, (1965) showed empirically that stock prices satisfy the principle of lack of memory, implying past price fluctuations cannot be used to predict future prices; while Lendasse and Bodt (2008), through the principle of efficient market hypothesis (EMH), explained that the future behaviour of stock prices can only be explained using current information. A number of recent papers described the use of Markov chains in predicting stock price fluctuations, these include Eseoghene, (2011), Christian and Timothy, (2014), Raheen and Ezeupe, (2014), and Bairagi and Kakaty, (2015]), to mention a few. In all these, little or no consideration has been given to the test of time homogeneity of the transition probability matrix.

In stochastic analysis, the Markov chain specifies a system of transitions of an entity from one state to another. Identifying the transition as a random process, the Markov dependency theory emphasizes “memory-less property” i.e. the future state (next step or position) of any process strictly depends on its current state but not its past sequence of experiences noticed over time. Aguilera et al. (1999) noted that daily stock price records do not conform to usual requirements of constant variance assumption in conventional statistical time series. It is indeed noticeable that there may be unusual volatilities, which are unaccounted for due to the assumption of stationary variance in stock prices given past trends. To surmount this problem, models classes specified under the Autoregressive Conditional Heteroskedastic (ARCH) and its Generalized forms (GARCH) make provisions for smoothing unusual volatilities.

Against the characteristics of price fluctuations and randomness which challenges application of some statistical time series models to stock price forecasting, it is explicit that stock price changes over time can be viewed as a stochastic process. Aguilera et al. (1999) and Hassan and Nath (2005) respectively employed Functional Principal Component Analysis (FPCA) and Hidden Markov Model (HMM) to forecast stock price trend based on non-stationary nature of the stochastic processes which generate the same financial prices. Zhang and Zhang (2009) also developed a stochastic stock price forecasting model using Markov chains.

Fama (1965) identified the highly stochastic nature of stock price behaviour. Bachelier (1914) proposed the theory of random walk to characterize the fluctuations in stock prices overtime. Fama

(1965) confirmed the empirical evidence of stock prices to satisfy the principle of random hypothesis; that a series of price changes has no memory, indicating past price dynamics cannot be used in forecasting the future price. According to efficient market hypothesis (EMH), security price changes can only be explained by the arrival of new information (Fama, 1965; 1970), which is quite challenging to predict (Lendasse, et al, 2008).

The determination of the long-run prospects of securities has proved to be quite a difficult challenge; as increasingly academic investigations have tended to agree with the notion that stock price movements are random and as such behaves in a dynamic and unpredictable manner (see, for example, Fielitz and Bhargava, 1973; Fielitz, 1969; Turner, Startz and Nelson, 1989; Hamilton, 1989; Obodos, 2005; and Idolor, 2009). Markov theory is seen to be relevant to the analysis of stock prices in two ways: (a) as a useful tool for making probabilistic statements about future stock price levels; and (b) as an extension of the random walk hypothesis. In this role, it constitutes an alternative to the more traditional regression forecasting techniques to which it is in some unique way superior in the analysis of stock price behaviour.

Markov theory is concerned with the transition of a system from one state to another. In this study, the model considered is that of a first-order Markov Chain. The particular Markov Chain studied here has a finite number of states and a finite number of points at which observations are made. In the analysis, we made use of standard methods, developed by Anderson and Goodman (1957), which was adopted and applied in Fielitz (1969 and 1971), Fielitz and Bhargava (1973), Obodos (2005), and Idolor (2009)

Since the stock market is a volatile market which has the random walk property, models which capture volatility would be expected to inform good predictions, as daily stock price records do not conform to usual requirements of constant variance in the common statistical time series. This study therefore, sought to predict the stock market trend of Dangote cement shares trading in the Nigerian Stock Exchange using Markov chain analysis. The specific objectives are (1) to study the trend of Dangote cement share prices in the Nigerian Stock Exchange; (2) to determine the Markov model for forecasting Dangote Cement share prices in the Nigerian Stock Exchange; and (3) to forecast Dangote Cement share prices in the Nigerian Stock Exchange. A simple test for the time homogeneity of the transition probability matrix is also derived.

2. Literature Review

2.1 Conceptual Framework

Stock market is a highly volatile market which generates arguments as to whether or not stock price can be predictive (McQueen and Thortey, 1991). By volatility, it means that stock prices fluctuate unexpectedly, with an investor making money when prices go up and losing in the event of a down turn. Bachelier (1914) looked at fluctuations in stock prices as a random walk. Fama (1965) showed empirically that stock prices satisfy the principle of lack of memory, implying past price fluctuations cannot be used to predict future prices, while Lendasse, DeBodt, Wertz and Verleysen (2008), through the principle of efficient market hypothesis, explained that the future behaviour of stock prices can only be explained using current information

Building on existing literature, we assume that stock price fluctuations exhibit Markov's dependency and time-homogeneity and we specify a three state Markov process (i.e. price decrease, no change and price increase) and advance the methodology for determining the mean return time for equity price increases and their respective limiting distributions using the generated state-transition matrices.

The usual assumption of Markov chain model obtains. These are stated as follows:

1. There is Markov dependency: that is, the future time $(t + 1)$, probability behaviour of the process is uniquely determined once the state of the system at the present time, t is given.
2. The Markov chain is time homogeneous. That is the one step transition probabilities are independent of time.

2.2 Theoretical framework

The stock markets in the recent past have become an integral part of the global economy. Any fluctuation in this market influences our personal and corporate financial lives, and the economic health of a country. The stock market has always been one of the most popular investments due to its high returns (Kuo, Lee & Lee, 1996; Hassan & Nath, 2005). However, there is always some risk to investment in the Stock market due to its unpredictable behaviour. So, an 'intelligent' prediction model for stock market forecasting would be highly desirable and would of wider interest. Reliable prediction of stock prices could offer enormous profit opportunities in reward and proactive risk management decisions. This quest has prompted researchers, in both industry and academia to find a way past the problems like volatility, seasonality and dependence on time, economies and rest of the market. The use of Markov processes in finance and economics is not new. Hamilton (1990) applies Goldfeld and Quandt's (1973) Markov switching regression in characterizing growth dynamics within an autoregressive process, and observes that an economy switches between two distinct phases of fast and slow growths in a manner governed by the outcome of a Markov process. Nefti (1984) applies a secondorder Markov process to US employment data and finds that the US economy transits between two states (rising and falling states) with respect to unemployment rates. Kim and Nelson (1998) and Kim, *et al*. (1998) also apply regime-switching models to stock returns from the US data. Chu, Santoni and Liu (1996) adopted a two-step approach to underpin stock return behaviour. First, they model stock return as a Markov switching process, and then estimate a volatility equation, given different return regimes derived in the first stage. Several theoretical and empirical studies were carried out in an effort to model and predict the market volatility patterns (Ibiwoye & Adeleke, 2008; Hamadu & Ibiwoye, 2010; Hamadu, 2014a; Hamadu, 2014b; Fama, 1965). Earlier, Kendal (1953) suggested that stock prices follow a simple random and changes in price are independent as well as the gains and losses. This study examines the Markovian characteristics of the Nigerian equity market. Markov processes are used to model systems with limited memory. They are used in many areas including actuarial science, financial engineering and modeling, resource management, communication systems, transportation networks and decision systems amongst others.

2.2.1 Markov Processes

Any variable whose value changes over time in an uncertain way is said to follow a stochastic process. A Markov process is a particular type of stochastic process where only the present value of a variable is relevant for predicting the future. The past history of the variable and the way that the present has emerged from the past are irrelevant. Stock prices are usually assumed to follow a Markov process (Hull, 2018).

A Markov chain is a sequence of experiments that consists of a finite number of states with some known probabilities P_{ij}, where P_{ij} is the probability of moving from state i to state j or simply put is stochastic process which depends on immediate outcome and not on history. It may be regarded as a series of transitions between different states, such that the probabilities associated with each transition depends only on the immediate proceeding state and not on how the process arrived at that state and the probabilities associated with the transitions between the states are constant with time.

When the present outcome is known, information about earlier trials does not affect probabilities of future events. The Markov chain model can then be said to be a sequence of consecutive trials such that

$$P\{X_n = j/X_{n-1} = i_{n-1}, \dots X_0 = i_0\} = P\{X_n = j/X_{n-1} = i_{n-1}\} \tag{1}$$

$P\{X_n = j\} = P_j^{(n)}$ is the absolute probability of outcome P_j, $j = 1, 2, 3, \dots$ is a system of events (actually set of outcomes at any trial) that are mutually exclusive.

A Markov chain is a model that tells us something about the probabilities of sequences of random variables, states, each of which can take on values from some set. A Markov chain makes a very strong assumptijon that if we want to predict the future in the sequence, all that matters is the current state. The state before the current state have no impact on the future except via the current state.

More formally, consider a sequence of state variables $q_1, q_2, \dots, q_j$. A Markov model embodies the Markov assumption on the probabilities of this sequence: that when predicting the future, the past doesn't matter, only the present.

Markov Assumption: $P(qi = a|q1 \dots qi - 1) = P(qi - a|qi - 1$ (2)

A Markov chain is useful when we need to compute a probability for a sequence of observable events. In many cases, however, the events we are interested in are hidden: we don't observe them directly. We call them hidden because they are not observed.

A first-order hidden Markov model instantiates two simplifying assumptions. First, as with a first-order Markov chain, the probability of a particular state depends only on the previous state:

$$P(qi|q1 \dots qi - 1) = P(qi|qi - 1) \tag{3}$$

Second, the probability of an output observation oi depends only on the state that produced the observation qi and not on any other state or any other observation:

Output Independence: $P(oi|q1 \dots qi \dots qTo i, \;\; \dots oi \;\; \dots oT) = P(oi|q1)$ (4)

That is to say, a Markov process is a particular type of stochastic process where only the present value of a variable is relevant for predicting the future. The occurrence of a future state in a Markov process depends on the immediately preceding state and only on it (Taha, 2001; Hull, 2018).

If $t_0 < t_1 < \dots t_n$ $(n = 0, 1 \; 2, \dots)$ represents points in time, the family of random variables $\{\xi t_n\}$ is a Markov process if it possesses the following Markovian property:

$P\ \{\xi t_n\} = X_n|\xi t_{n-1} = X_{n-1}. . ., \xi t_0 = X_0 = P\ \{\xi t_n\} = X_n|\xi t_{n-1} = X_{n-1}\}$ (5)

For all possible values of $\xi t_0, \xi t_1, \ldots\ \xi t_n$

The probability $PX_n,\, X_n = P\ \{\xi_{tn} = X_n|\xi_{tn} = X_{n-1}\}$ is called the transition probability **(**a term primarily used in mathematics and is used to describe actions and reactions to what is called the "Markov Chain."

It represents the conditional probability of the system being in X_n at t_n, given it was in X_{n-1} at t_{n-1} (with X representing the states and t the time). This probability is also referred to as the one-step transition because it describes the system between t_{n-1} and t_n. An m-step transition probability is thus defined by:

$P_{xn}, X_{n+m} = P\{\xi_{tn+m} = X_{n+m}|\xi_{tn} = X_n\}$ (6)

Formally, a stochastic process includes the description of a probability space (Ω, F, P) and a family of random variables (indexed by $t\ \in [0, \infty)$)

$X(t): \omega \to X(t)(\omega)\ \epsilon\ N_0$ (7)

For the stochastic process to be a Markov chain the distribution of $\{X(t)\}_{t\geq 0}$ must satisfy some mathematical conditions that it is hard to state and verify formally.

The stochastic process $\{X\ (t),\ t \in T\}$ is said to exhibit Markov dependence if for a finite (or countable infinite) set of points $(t_0, t_1, . . . , t_n,\ t), t_0 < t_2 < . . . < t_n < t$ where $t, t, \in T\ (r = 0, 1, 2, . . . n)$.

$$P(X(t) \leq x|X(t_n) = x_n, \qquad X(t_{n-1}) = x_{n-1}, . . ., X(t_n) = x_0)$$

$= P[X(t) \leq x|X(t_n) = x_n] = F[X_n,\ \text{x}: t_n,\ t]$ (8)

From the property given by equation (8), the following relation suffices:

$F(X_n,\ x;\ t_n, t) = \int_{y\in S} F(y, x\!: \tau, t) dF(X_n,\ \ y\!: t_n, t)$ (9)

Where $t_n < \tau < t$ and S is the state space of the process {X (t)}.
When the stochastic process has discrete state and parameter space, (3) takes the following form: for $n > n_1 > n_2\ . . . > \in n_k$ and $n, n, \in T\ (r = 1, 2,\ . . . k)$

$P(X_n = j|X_{n1} = i_1,\ . . . X_{nk} = i_k$

$= P(X_n = j|X_n = i_1) = P_{ij}^{nk-n}$ (10)

A stochastic process with discrete state and parameter space which exhibits Markov dependency as in equation (10) is known as Markov property, for $n_k < r <$ we get

$$P_{ij}^{nk-n} = P(X_n = j | X_{nk} = i$$
$$= \sum_{m \in S} P(X_n = j | X_r = m) P(X_r = m | X_{nk} = i)$$

$$= \sum_{m \in S} P_{mj}^{(n_k, r)} \quad (11)$$

Equation (9) and (11) are known as the Chapman-Kolmogorov equation for the process.

2.3 Basic Concepts

2.3.1 n-step transition probability matrix and n-step transition probability

An important class of Markov chain model is that of which the transition probabilities are independent of n, we have $P\{X_n = j / X_{n-1} = i\} = P_{ij}$ which is a homogenous Markov chain where the order of the subscripts in P_{ij} corresponds to the direction of the transition i.e. $i \to j$. Hence we have $\sum_{i=1}^{n} P_{ij} = 1$ and $P_{ij} \geq 0$. Since for any fixed i, the transition probability P_{ij} will form a probability distribution. If the limiting distribution of x_n as n$\to \infty$ exists, the transition probabilities are most conveniently handled in matrix form as $P = P_{ij,}$ i.e.
This is referred to as the transition matrix, which depends on the number of states involved and may be finite or infinite (Hamilton, 1989; Michael, 2005).
The absolute probabilities at any stage where n is greater than unity is determined by the used of n-step transition probabilities; i.e.
In matrix terms, let p be the transition matrix of the Markov chain, then:

$P^1 = PP^2 \ (for\ n = 1)$

Also $P^2 = PP^2 = P(PP^{(0)} = P^2 P^{(0)} \ (for\ n = 2)$

And in general $P^{(0)} = P^n P^{(0)}$

If P is the transition probability matrix of a Markov chain $\{X_{n,} n = 0, 1, 2,, \ , \ , \ ,\}$ with state space S, then the elements of P^n $(P\ raised\ to\ the\ power), P_{ij}^{(n)} i, j \in S$ are the n-step transition probabilities where $P_{ij}^{(n)}$ is the probability that the process will be in state j and the nth step starting from state i.
The statement above can clearly be shown from the Chapman-Kolmogorov equation (11) as follows; for a given r and s, write:

$$P_{ij}^{(s+r)} = \sum_{k \in s} P_{ik}^{(r)} P_{kj}^{(S)} \quad (12)$$

Set $r = 1, s = 1$ in the above equation to get

$$P_{ij}^{(2)} = \sum_{j \in s} P_{jk} P_{kj} \quad (13)$$

Clearly, $P_{ij}^{(2)}$ is the $(i,j)th$ element of the matrix product $PxP = P^2$. Now suppose $P_{ij}^{(r)}(r = 3, 4,. . . , n)$ is the $(i,j)th$ of $P^{"}$ then by the Kolmogorov equation, the

$$P_{ij}^{(r+1)} = \sum_{k\in S} P_{ik}^{(r)} P_{kj} \quad (14)$$

which again can be seen as the $(i,j)th$ element of the matrix product $P'P = P^{r+1}$. Hence by induction, $P_{ij}^{(n)}$ is the $(i,j)th$ of P^n $n = 2, 3,. . . .$
to specify the model, the underlying assumption is stated about the identified n-step transition probability (stating without proof).
The transition probability matrix is accessible with existing state communication. Further, there exists recurrence and transience of states. States are also assumed to be irreducible and belong to one class with the same period which we take on the value. Thus the states are periodic.

2.3.2 Derivation of the Three State transition Matrix

The transition matrix we would require involves three states only The states are the chances that a stock decreases, that it remains the same (unchanged) and that it increases. We state the three states as follows:
D = share price decreases
U = share price remains the same
I = share price increases.

Matrix of transition probabilities provides a precise description of the behavior of a Markov chain. Each element in the matrix represents the probability of the transition from a particular state to the next state. The transition probabilities are usually determined empirically, that is based solely on experiment and observation rather than theory. In another way, relying or based on practical experience without reference to scientific principles. Historical data collected can be translated to probability that constitute the Markov matrix of probabilities.

2.3.3 Recurrence and transience of state

Let X_t be a Markov chain with state space S, then the probability of thr first transition to state j at the t^{th} step starting from state i is

$$f_{ij}^{(t)} = P[X_t = j,\ X_t = j; r = 1\ . . ., t-1|X_0 = i] \quad (15)$$

The probability that the chain ever returns to state j is:

$$f_{ij} = \sum_{t=1}^{\infty} f_{ij}(t)$$

and $u_{ij} = \sum_{t=1}^{\infty} t f_{ij}^{(t)}$ is the expected value of first passage time. Further, if $i = j$, then;

$$f_{ij}^{(t)} = P|X_t = i,\ \ X_t = i;\ \ r = 1, 2, 3, . . . \ , t-1|X_t = i] \quad (16)$$

and $\mu_i = \mu_j = \sum_{t=1}^{\infty} t f_i^{(t)}$is the mean recurrence time of state i if state i is recurrence.
A state i is said to be recurrence (persistent) if and only if, stating from state t, eventual return to this state is certain. Thus, state i is recurrent if and only if

$$f_{it}^{*} = \sum_{t=1}^{\infty} f_{it}^{(t)} = 1 \quad (17)$$

A state t is said to be transient if and only if, stating from state i, there is a positive probability that the process may not eventually return to this state. This means $f_{it}'' < 1$.

2.4 Empirical Review

Dallah and Adeleke (2018) investigate the Markovian characteristics of the Nigeria Stock Market using weekly data on All Share Index (ASI) market, 30- Index and five sub-sectors of Nigerian stock exchange from October 4, 2013 to September 30, 2016. The Chapman-Kolmogorov's principles of handling transition probabilities and limiting distributions methods were employed for predicting future market behaviour. Generally, the findings established the volatile nature of the market and its rapid tendency for deterioration.

Fitriyanto and Lestari (2018) apply Markov Chain in PT HM Sampoerna stock price. A Markov Chain model was determined based on probability transition matrix and initial state vector. In the long run, the model predicted that PT HM Sampoerna share prices with probability 0.09, 0.40, 0.46, and 0.05 respectively.

Esbond and Saporu (2017) used a 5-state Markov chain model to model the behaviour of daily stock price on the floor of the stock market. A simple test for the time homogeneity of the arising transition probability matrix is proposed. The results show that it is best to buy stock when price is at high depreciation state, and sell within two days when next the price rises to the high appreciation state. The use of these criteria is recommended.

Aparna, and Kakaty (2017), in predicting the future market price of potatoes applied the Markov Chain model. The data related to the market price of potatoes from 4th June 2014 to 21st April 2017 i.e. for 478 days were collected to predict the future price interval for fifteen consecutive days. Using the transition probability matrix and initial state vector the prediction for short period were made. The results obtained from the analysis were indistinguishable from real situation.

Okonta, Elem-Uche and Madu (2017) focused primarily on the application of Markovian analysis on Nigerian stock market weekly returns with the aim of describing the distribution of the market returns, investigating its trend, cross-checking if a current market return depends on its preceding market return and the time independence of the transition probabilities, investigating the probability of an investor earning return and the expected time a market return remains in a state before moving to another state. The study also showed when the very short time is considered, information in the past data is not fully reflected in present prices.

Raheem and Ezepue, (2016) present in their study an alternative approach to determining and predicting the fluctuations in the daily prices and stock returns of a first generation bank in the

Nigerian Stock Market (NSM). The findings further reveal a minimum trading cycle of 7 days in February and a maximum cycle of 18days in the months of May and October. The paper provides useful insights not only on the durations of returns in the three states, but on the Markovian transition probabilities among pairs of states which have implications for how investors could trade and invest in the bank stock or in a portfolio with bank stocks, if the same approach is used to characterize the returns dynamics of other banks in the NSM.

Ezugwu and Igbinosun (2016) provide a review of Markov decision processes and investigate its suitability for solutions to portfolio allocation problems under vendor managed inventory in an uncertain market environment. It was observed that the optimal policy is expected to always short the stock when in state 0 because of its large return. However, while the return is not as large as in state 0, the probability of staying in state 2 is high enough that the vendor should long the stock because he expects high reward for several periods. We also obtained the expected reward for each state every ten iterations using a discount factor of $\lambda = 95.0$. In spite of the small state/action spaces, the investor is able to optimize its reward by the use of Markov decision process.

From the above review, it comes out that Markov Chain model has been widely applied in predicting stock market trend. In many applications, it has been applied in predicting stock index for a group of stock but little for a single stock. Moreover, the model has had limited application in emerging stock markets. This study therefore sought to apply Markov Chain model to study the trend of Dangote Cement and Dangote Four (individually) shares trading in the Nigerian Stock Exchange, as an emerging market

3. Methodology

3.1 Design

This study used case study design. The Dangote Cement stock was floated and started trading in the Nigerian Stock Exchange in November 4, 1992. The case study designed was therefore designed longitudinal in nature covering two years period.

3.2. Data source

Investors' decisions with respect to buying or selling of equity are potentially influenced by the daily information provided by stock prices. Thus, stock prices are fundamental index for analyzing and forecasting the stock market. They are less of human's errors since they represent actual trading activities. In this study, the researcher employs the data on the stock of Dangote Cement company, which is continuously traded on floor of Nigerian Stock Exchange (NSE) over the period 1st January 2017 to 31st December 2018. Specifically, our observation consists 464 trading days.

3.3 Model Specification

This study uses the Markovian characteristics of the stochastic processes of the market and stocks returns. The methodology adopted the Chapman Kolmogorov's principles of handling transition matrices and limiting distributions. The returns are assumed to be a first-order Markov process $\{X(t)|t \in T\}$ for any $t_0 < t_1 < \ldots < t_n$ the conditional cumulative distribution function of $X(t_n)$ for a sample values of $X(t_0), X(t_1), \ldots, X(t_{n-1})$ depends only on $X(t_{n-1})$. which can be written as:

$P[X(t_n) \leq x_n | X(t_{n-1}) = x_{n-1}, X(t_{n-2}) = x_{n-2}, . . . , X(t_0) = x_0]$

$= P[X(t_n) \leq x_n | X(t_{n-1}) = x_{n-1}$ (3.1)

Thus, given the present state of the process, the future state is independent of the past.In the case of the strong Markov property, for all *t*, the process {*X* (*t*+*s*) - *X* (*t*) |s≥ 0} has the same distribution as the process {*X*(*s*)| *s* ≥ 0} and is independent of $\{X(s|0 \leq s \leq t\}$. Thus, when the state of process is known at time *t*, the governing probability of the future change of state of the process will be determined as if the process started at time *t*, independently of the history of the process up to time *t* (Ibe, 2009),while the time *t*is a constant. The strong Markov property allows for the replacement of the fixed time t with a non-constant random time

3.4 Discrete time Markov chain

This study considers the discretized –time process $\{X_t, t = 0,\ 1.\ 2, . . . \}$ for all $i,\ j,\ n,\ m$, given $P[X_t = j | X_{t-1} = i, X_{t-2} = n, . . ., X_0 = m] = P[X_t = j | X_{t-1} = i] = P_{ijt}$.
Where P_{ijt}.is the state transition probability, which is the conditional probability that the process will be in a state j at time t immediately after the next transition, given that it is in a state i at time $t - 1$. The Markov chain that strictly follows this rule is a non-homogeneous Markov chain. However, this study considers homogeneous Markov chains in which:
$P_{ijt} = P_{ij.}$.
Implying that:

$$P[X_t = j | X_{t-1} = i,\ X_{t-2} = n, . . ., X_0 = m] = P[X_t = j | X_{t-1} = i] = P_{ij} \quad (3.2)$$

Where $0 \leq P_{ij} \leq 1$, and $\sum j P_{ij} = 1, i = 1,\ 2, ... n$

The P_{ij}, are usually displayed as a square matrix P, where P_{ij} is the entry in the i^{th} row and j^{th} column:

3.5 Determination of initial state vector

Each closing day is taken as a discrete time unit, then the closing share prices are divided into three states namely Decreases (D), Unchanged (U) and Increases (I). Let *x1* = D, *x2*= U and *x3*= I, where *xi* are the number of observations for the share prices in the named states gathered over the period of study, then the state space is E(*x1*, *x2*, *x3*). State probability is the possibility size of emergence of a variety of states. State vector is denoted by *n(i)*=(*p1, p2,……., pn*) where *i*= 1,2,….n, P_j is the probability of x_j, j=1,2,…,n. Since there are 768 trading days in the four year period of study, the observations for D, U and I amounts to 464. So the probability of each state will be as follows:

3.6 Establishment of the three state transition matrix

The transition matrix for this study involves three states only as Dangote cement stock assume basically three states. The states are the chances that a stock decreases, that it remains the same (unchanged) and that it increases. The three states are stated as follow:

D = share price decreases
U = share price remains the same (Unchanged)
I = share price increases.

Transition Probability Matrix provides a precise description of the behavior of a Markov chain. Each element in the matrix represents the probability of the transition from a particular state to the next state. The transition probabilities are determined empirically, that is based solely on experiment and observation rather than theory. In other words, relying or based on practical experience without reference to scientific principles.

This study is based on historical daily closing prices of shares quoted on the Nigerian Stock Exchange. The data on share prices of the study was collected from the daily list published by the Nigerian Stock Exchange (also the daily Nation newspaper) from January,2018 to December, 2019. The transitions from one state to another (that is the share price movement pattern, which could be that a decrease in price can be followed by another decrease or a decrease is followed by unchanged or a decrease followed by an increase etc) was observed from the data collected and the result for the period (2 years) under study was compiled in Tables 1.

Each entryP_{ij} in the table refers to the number of times a transition has occurred from state i to state j. Consequently, from the share price movement compiled, the transition probabilities were computed to obtain transition matrix for Dangote Cement Stock as shown in Tables 1. The states 1, 2 and 3 represent the Price, Unchanged, Increases and Decreases respectively. The probability transition matrix P_{ij} is formed by dividing each element in every row by the sum of each row.

3.7 The n-step Transition matrix

The absolute probabilities at any stage where n is greater than unity is determined by the used of n-step transition probabilities. This is a higher order transition probability $P_{ij}(n)$ of the transition matrix P_{ij}. The n-step matrix shows the behavior of share prices n-steps later. The elements of this matrix represent the probabilities that an object in a given state will be in the next state n-steps later. These repeated transitions are used to evaluate whether the transition probabilities converge over repeated iterations

This results in steady state probabilities which shows the probabilities of the shares prices increasing, remaining unchanged or decreasing regardless of the share's most recent changes in daily closing price.

4. Analysis, Results and Discussion

4.1 Determination of Transition and Transition Probability Matrices

A close observation of the prices of these companies over the study period shows that they evolve through three different states of transition. The possible movements at the end of each trading day by the companies' prices include either: increasing, decreasing or unchanging. Therefore, for the purpose of establishing a transition probability matrix, these three different movements are considered as three different states in the Markov chain. We present the transition matrix of Dangote Cement in table 1.

Table 1-Transition Matrix of Dangote Cement

	Unchanging	Increasing	Decreasing
Unchanging	28	45	42
Increasing	73	2	85
Decreasing	62	117	10

Source: Author

The table above shows that Dangote Cement stock traded for 464 days in the period January 1st 2017 to December 31st 2018. It is clearly evident that the company's prices were up for 160 days, unchanged for 115 days and down for 189 days.

Transition Matrix for Dangote Cement

$$P_{Dangcem} = \begin{pmatrix} 0.24 & 0.39 & 0.37 \\ 0.46 & 0.01 & 0.53 \\ 0.33 & 0.62 & 0.05 \end{pmatrix}$$

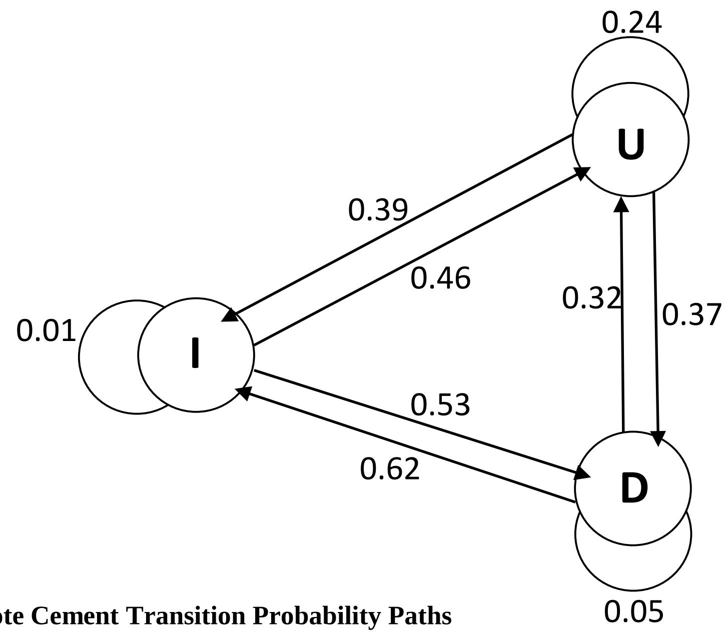

Figure 1-Dangote Cement Transition Probability Paths

4.2 Transition Probability Vector

The initial state vector is the probability vector that the stock prices would exhibit three states, unchanging, increasing and decreasing. Let us denote this vector by θ_0, and then it can be defined as.

$$\theta_0 = [\theta_1 \theta_2 \theta_3]$$

Where θ_1, θ_2 and θ_3 are the probabilities that the stock prices would stay unchanged, increased and decreased respectively. In view of this, we can now obtain the values of these probabilities by computing their respective weights as follows for each of the stock.

Initial Weight for Each of the three States of Dangote Cement

$$\theta_1 = \frac{115}{464} = 0.25$$

$$\theta_2 = \frac{160}{464} = 0.34$$

$$\theta_3 = \frac{189}{464} = 0.41$$

The Correspondent Initial Probability Vector

$$\theta_0 = \begin{pmatrix} 0.25 & 0.34 & 0.41 \end{pmatrix}$$

4.3 Results

The main results of this study are based on forecasting the stock prices of Dangote Cement company as at the end of 465th day and long-range behaviors of each of the two stocks. We first, compute the probabilities of forecasting the stock prices at the end of 465th day for each of the companies. According to the Markovian chain, these vector of probabilities can be obtained by multiplying initial state vector (θ_0) and probability matrix (p) together. That is $\theta_{i+1} = \theta_i * p^j \quad ; i = 0,1,2,... \quad ; j = 1,2,...$ therefore, the state probabilities for each of the companies can be computed as follows.

State Probability for Dangote Cement as at the end of 245th day (h_1)

$$h_1 = \theta_0 * P_{Dangcem} = \begin{pmatrix} 0.25 & 0.34 & 0.41 \end{pmatrix} \begin{pmatrix} 0.24 & 0.39 & 0.37 \\ 0.46 & 0.01 & 0.53 \\ 0.33 & 0.62 & 0.05 \end{pmatrix}$$

By applying mmult in excel-spread sheet, we have.

$h_1 = (0.3517 \quad 0.3551 \quad 0.2932)$

These results show that at the end of the 465th day the prices of Dangote Cement will remained unchanged with the probability of 35 percent, increase with the probability of 36 percent, while the probability that the prices will decline is 29 percent. Let us compute the state probabilities for the company as at 466th day (h_2). Where:

$$h_2 = \theta_0 * (P_{Dangcem})^\wedge 2 = \begin{pmatrix} 0.25 & 0.34 & 0.41 \end{pmatrix} \begin{pmatrix} 0.24 & 0.39 & 0.37 \\ 0.46 & 0.01 & 0.53 \\ 0.33 & 0.62 & 0.05 \end{pmatrix}^{\wedge 2}$$

$$h_2 = (0.34451 \quad 0.322498 \quad 0.332992)$$

The results further reveal that at the end of 466th day the probability that the company's stock price will decline increases to 33 percent, the probability that the stock price will increase or remain unchanged decrease to 34 and 32 percent respectively. These results provide information that investing in this stock is not a good long-term adventure since the possibility of price decrease increases with time.

We can now proceed to examine the long-range behavior of these companies. Long-range behavior of stock is very important since investors are highly interested in taking decision today that could change the fortune of their expect future. To achieve this, we compute n-step transition probability matrix or high-order transition probability matrix (P_n) for the company is as follows.

High-order Transition Probability Matrix of Dangote Cement

The high-order transition matrix of Dangote Cement stock price shows that the matrix converges to steady state 11th day from the 465 trading days, suggesting that in the long run, irrespective of the initial states, the company's prices remain stable with probability of 34 percent, increase with probability of 34 percent, and then decline with probability of 32 percent. However, if the price of Dangote Cement starts in certain state with initial state vector (θ_0), then the probability that the company will increase, remains same or decrease at a particular tradingday can be determined by multiplying the initial transition vector by the high-order probability as given below.

$$\theta_0 * P_{11} = \begin{pmatrix} 0.25 & 0.34 & 0.41 \end{pmatrix} \begin{pmatrix} 0.34 & 0.34 & 0.32 \\ 0.34 & 0.34 & 0.32 \\ 0.34 & 0.34 & 0.32 \end{pmatrix}$$

$$\theta_0 * P_{11} = (0.34 \quad 0.34 \quad 0.32)$$

This shows that in the long run equilibrium state, the probabilities that the stock price of Dangote Cement Company will remained stable, increase and decrease are 0.34, 0.34 and 0.32 respectively.

5. Conclusion

It is concluded that the daily closing Dangote cement share prices had generally a bullish and bearish trend alternating, though with an initial sideway trend, indicating volatility of prices. The objective was to determine the Markov model for forecasting Dangote cement share prices in the Nigerian Stock Exchange, it was concluded that the derived initial state vectors and the transition matrices could be used to predict the state of Dangote Cement prices accurately. Additionally, the convergence of transition matrices to a steady state implying ergodicity that is a characteristic of stock market makes the model applicable. Finally, in the long run, irrespective of the current state of shares prices, the model predicted that the Dngote Cement share prices will depreciate with probability of 29 percent, maintain value with the probability of 35 percent, or appreciate with approximate 36 percent.

The Markov chain prediction method is purely a probability forecasting method as the predicted results is simply expressed probability of certain state of stock or shares prices in the future rather than the absolute state. But because it has no after-effect, using this method to analyze and predict the stock market daily closing prices is relatively more effective under the market mechanism. This study shows how Markov model fits the data and is able to predict trend due to its memoryless property and random walk capability, in that each state can be reached directly by every other state in the transition matrix, consequently giving good results.

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